A Dimension-Reduced Cosine-Expansion Method for Solving Multivariate Expectations

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The Fourier-cosine expansion method for density recovery and option pricing, or the COS method in short, was first proposed by Fang and Oosterlee in [1] to price European options, and since then, it has been widely used in the field of financial mathematics. For the purpose of option valuation, extensions have been made, for example, to pricing Barrier and American options [2, 3] and Asian options [4] as well as to the 2D-COS method [5]; For the purpose of risk quantification, it has been applied and adjusted to fast computing measures like Value-at-Risk (VaR) and Expected Shortfall (ES) [6] and lately to the calculation of Euler allocation measures [7]; In the field of insurance risk theory, the COS method has been applied in the recovery of various density functions, such as the ruin probability [8], the discounted density function of the deficit at ruin [9], the finite time ruin probabilities [10], etc. In this talk, we will present our recent research work on the COS method for efficient calculation of multivariate expectations. Multivariate expectation is the mathematical problem common to various fields related to the probability theory. Very often there exists no analytical solution and one has to rely on numerical methods. Our key insight is that, in many applications in finance, the original multivariate expectation problem can be reformulated as a univariate problem which is easier to solve in the Fourier domain. To be more specific, the original problem can be transformed to solving the characteristic function (ch.f.) of the combined dynamics of the multiple random variables involved. To compute this ch.f., we developed a dimension-reduced cosine-expansion, via applying Carnonic Polyadic Decomposition (CPD) to the Fourier-cosine expansion coefficient tensor. This method is thus named COS-CPD method. It avoids the curse-of-dimension in the on-the-fly calculations and is tested to be very efficient and stable. We will illustrate the application of the COS-CPD method using the example of Potential Future Exposure (PFE) and XVA calculations for netting pools of Over-the-Counter (OTC) derivatives.