Empirical and theoretical analysis of profit and loss allocations

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The decomposition of the investment profit and loss (p&l) for each business year into different risk factors (e.g., interest rates, credit spreads, foreign exchange rate etc.) is a task that is regulatory required, e.g., by Solvency 2. Three different decompositions are prevalent: one-at-a-time (OAT), sequential updating (SU) and average sequential updating (ASU) decompositions. The SU and the ASU decompositions explain the p&l fully, i.e. are exact. However, the OAT decomposition generates some unexplained p&l. The SU decomposition depends on the update order of the risk factors, i.e., if there are d risk factors, there are $d!$ SU decompositions. The ASU decomposition is defined by the average over all $d!$ SU decompositions. The three decompositions can be defined on different sub-intervals using annually, quarterly, monthly, weekly or daily data. In this talk, using financial market data from 2003 till 2022, we empirically quantify: the unexplained p&l of the OAT decomposition; the dependence of the SU decomposition on the update order; and how much the three decomposition principles depend on the size of the sub-intervals. We will see that the ASU decomposition is the most useful decomposition in practice. In the second part of the talk, we derive a generalized Ito's formula. We obtain a new family of decompositions from the generalized Ito's formula, called Ito decomposition, and we show that this family contains the OAT, SU and ASU decompositions as limiting cases. We show that there is only one Ito decomposition, which satisfies three axioms: exactness, normalization and symmetry. This Ito decomposition is called IASU decomposition and is the limiting case of the ASU decomposition. The IASU and ASU decomposition scale like $O(d!)$, i.e., they suffer from the curse of dimensionality. In the limiting case, we show that it is possible to break the curse of dimensionality: asymptotically, the ASU decomposition can be approximated by just two SU decompositions if the risk factors do not have simultaneous jumps. Therefore, analyzing the limit of the ASU decomposition has also practical relevance.