

Efficient Collective Investment with Limited Expected Loss: Pareto-optimal Wealth Sharing and Risk Allocation

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This paper investigates a collective investment problem under a limited expected loss (LEL) constraint in a complete market setting. We propose a novel LEL sharing rule as an alternative to widely used proportional sharing rules, demonstrating a direct correspondence between LEL sharing rules and constrained Pareto-optimal sharing rules. Under the financial fairness condition, we derive a unique LEL sharing rule through a straightforward fixed-point iteration. This rule connects each participant's share to the optimal terminal wealth in individual LEL-constrained scenarios, implying an intriguing recomposition effect. By evaluating the participants' tail risks via expected relative loss risk metrics, we formulate a novel Pareto-optimal risk allocation for such constrained collective risk management. Our numerical analysis affirms the theoretical findings and underscores the positive influence of the LEL constraint among prevalent proportional sharing rules, emphasizing the importance of risk control in practical scenarios. We observe that in contrast to LEL sharing rules, proportional sharing rules considerably lower reward-to-risk ratios for most collective members. This might discourage individual participation in the collective under an LEL constraint, conveying a cautionary message to collective planners regarding the determination of sharing rules.