Optimal Smooth Consumption and its Trade-Offs

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We investigate different ways to design smooth pension products based on solutions to optimal consumption and investment problems. The smoothness of a consumption process can be studied from both a pathwise (measured in terms of quadratic variation) and a pointwise (measured in terms of variance) point of view, and we conclude that introducing one type of smoothing does not necessarily improve the other type of smoothing. Thus, care must be taken when designing smooth pension products. Focusing on pathwise smoothness without disregarding pointwise smoothness, we provide both a qualitative and a quantitative discussion of the trade-offs involved. In the qualitative discussion, we find that to increase smoothness, it is necessary to reduce the starting value, the drift of consumption, or the level of terminal wealth. For the quantitative discussion, we set up an optimal consumption and investment problem, where the first control is the proportion of wealth invested into the risky asset, but the second control is not the con- sumption process itself. Instead, we use the drift and volatility of consumption as controls. The objective is to minimize the quadratic distance to a target drift and volatility while introducing a penalty for the volatility. We find explicit solutions to this problem using classic dynamic pro- gramming methods and use them to study the three trade-offs theoretically and numerically. All three approaches result in both pointwise and pathwise smoothing compared to the target, but reducing the drift yields better pointwise smoothing for similar levels of pathwise smoothing.