A numerical solution for European option pricing under 2-dimensional jump-diffusion processes

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A numerical approach for approximating the value of European options will be presented, assuming that the dynamics of the underlying asset prices are described by a 2-dimensional jump-diffusion process. Following [1], the jump component is obtained by subordination of a 2-dimensional Brownian motion with a 1-dimensional tempered stable subordinator, as described in [2]. This yields an infinite activity process and, therefore, gives rise to an integral of a singular function in the PIDE (Partial-Integro-Differential equation) that holds for the option value function. The Variance Gamma and the Normal Inverse Gamma processes are the most famous special cases of this type of process.

Using the method of lines for solving the PIDE, the numerical scheme is divided into two general steps: the spatial discretization, in which the (spatial) integral-differential operator is replaced with a finite-difference version, converting the PIDE to a system of linear ODEs, and next the temporal discretization, in which this system is numerically solved by a suitable time-stepping method.

A special feature of the numerical approach under consideration, that is inspired by [3] and [4], is the fact that jumps smaller than a given threshold are replaced by a diffusive term. This makes it possible to remove the singularity from the integral term, and hence, to resort in principle to well-known numerical schemes for finite activity Lévy processes. Furthermore, the integral term in the PIDE corresponds to a dense matrix in the system of linear ODEs. To avoid the inversion of this matrix, a second-order IMEX (implicit-explicit) time-stepping method is proposed, which allows for an efficient evaluation by FFT and interpolation.

References:

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